Detection of Equipment Faults in Automatically Controlled Processes

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Based on the UBET, a new strategy for identifying faulty equipment for dynamic chemical processes under automatic control is presented. The strategy is designed to distinguish between measurement biases, controller biases, and process leaks. For illustration purposes, application is given to a level control process under pseudo-steady state. This approach was inspired by the work of Rollins and Devanathan to identify biased measurements under dynamic conditions. Advantages of this method are that it is not computationally intensive, can accurately detect and specifically identify the type of fault, and can accurately determine the time when the fault occurs.

Introduction

The accurate identification of faulty measurement equipment or a process leak is an important task in a process (Willsky and Jones, 1974; Willsky, 1976; Mah et al., 1976). Rollins and Davis (1992) presented the unbiased estimation technique (UBET) for identifying large systematic errors (e.g., measurement biases). However, at that time the UBET was limited to steady-state conditions. Rollins and Davis (1993) extended the UBET to pseudo-steady-state conditions. Narasimhan and Mah (1988) made pioneering progress in dynamic fault detection using a generalized likelihood ratio (GLR) statistic with serial compensation to identify various types of equipment faults in a closed-loop process in pseudo-steady state (also see Narasimhan and Mah, 1987). Upon examination of their approach, however, we have identified the following drawbacks: (1) estimation of state variables via Kalman filtering is computationally intensive; (2) test requires accurate estimation of time of occurrence (TOC) of the fault; and (3) test is sensitive to the value of delay time for identification.

The objective of our research was to develop an accurate fault-detection strategy for automatic processes in pseudo-steady state without the aforementioned drawbacks. Our UBET-based strategy is demonstrated on the same level control process used by Narasimhan and Mah (1988), and has the potential to achieve high identification accuracy and unbiased estimators. The method to develop linear discrete process models from the process differential equations follows the work of Rollins and Devanathan (1993, 1994). Based on the discrete process model, four measurement models are developed incorporating a term to represent pseudo-steady-state process variability. A new global test (GT) and a new identification test are proposed for detecting and identifying a biased measurement, a biased controller, or a process leak.

Measurement Model and Process Model

It is unlikely that the process measurements will vary due to measurement error alone, since changing process conditions will also contribute to the variability of process variables. Here, both sources of variability (measurement and process variability) are considered. For convenience, we are assuming that serial correlation does not exist. In situations where serial correlation is important, one would need to model it. The general measurement model that takes into account process deviation, measurement bias, and random error is (Figure 1)

$$y_{ji} = y_{ji}^* + \delta_j + \epsilon_{ji} = \mu_j + \lambda_{ji} + \delta_j + \epsilon_{ji},$$
$$\text{for} \quad i = 1, 2, \ldots, M \quad \text{and} \quad j = 1, 2, \ldots, p, \qquad (1)$$

where $y_{ji}^* = \mu_j + \lambda_{ji}$, $E[y_{ji}] = \mu_j + \delta_j$, $e_{ji} \sim N(0, \sigma_j^2)$, $\lambda_{ji} \sim N(0, s_{\lambda j}^2)$, $\mathrm{COV}(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$, $\mathrm{COV}(\lambda_i, \lambda_j) = 0$ for $i \neq j$, and $\mathrm{COV}(\epsilon_i, \lambda_j) = 0$ for all i, j.

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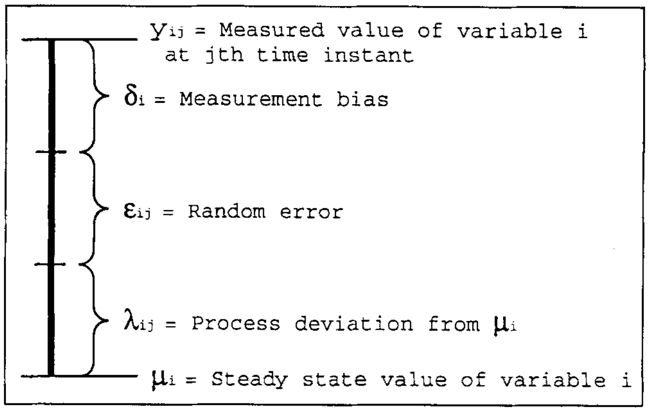

Figure 1. Error sources for measurement models under pseudo-steady-state conditions.

This model is similar to the one given by Rollins and Davis (1993) except for the addition of λ_{ji}. The process model for the level control process in Narasimhan and Mah (1988) (shown in Figure 1) is developed using deviation variables. We assume the random error and the measured process deviation variables h', X', and F' are normally distributed with mean 0 and known variances. From the process differential equation, we obtained a set of linear discrete equations (Cheng, 1994):

$$dh_i^* - bh_{i-1}^* + k_{X,2}X_i^* - F_i^* = 0 \tag{2}$$

$$X_i^* = ch_i^*, \tag{3}$$

where d, b, c, and $k_{X,2}$ are given constants, and h_i^*, X_i^*, and F_i^* are the unknown true values of h_i', X_i', and F_i' at time instant i. Modifying Eqs. 2 and 3 to account for process leaks and controller bias, we have

$$dh_i^* - bh_{i-1}^* + k_{X,2}X_i^* - F_i^* = -\lambda \tag{4}$$

$$X_i^* = ch_i^* + B. \tag{5}$$

Test Measurement Models

In order to detect and identify any equipment fault, four test measurement models are derived based on the two discrete process models. These four models are used in the proposed strategy that detects the existence of faults and identifies either a bias in h', X', F', or bias in the controller, or a leak in the process. Table 1 contains the summary information for these models.

Transformed Test Models

The test measurement models were transformed for statistical testing purposes. The transformed variable for Model I is

$$T_{U,i} = Q^{1\times 6}U_i^{6\times 1} = Q^{1\times 6}U_i^{*6\times 1} + Q^{1\times 6}\boldsymbol{\delta}_{U,i}^{6\times 1} + Q^{1\times 6}E_{U,i}^{6\times 1}$$
$$= -\gamma + Q^{1\times 6}\boldsymbol{\delta}_{U,i}^{6\times 1} + Q^{1\times 6}E_{U,i}^{6\times 1}, \tag{6}$$

since $Q^{1\times 6}Ui^{*6\times 1} = -\gamma$ is the model in Table 1. It follows that

Table 1. Test Measurement Models

Model	Measurement Model
Model I: Eq. 4	
$dh_i^* - bh_{i-1}^* + k_{X,2}X_i^* - F_i^* = Q^{1\times 6}U_i^{*6\times 1} = -\gamma$	$U_i^{6\times 1} = U_i^{*6\times 1} + \boldsymbol{\delta}_{U,i}^{6\times 1} + E_{U,i}^{6\times 1}$
where $i \geq 2$	$U_i^{*6\times 1} \sim N_6(\boldsymbol{\mu}_U^{6\times 1}, \Sigma_{\lambda U}^{6\times 6})$
$Q^{1\times 6} = [d, k_{X,2}, -1, -b, 0, 0]$	$E_{U,i}^{6\times 1} \sim N_6(0^{6\times 1}, \Sigma_{EU}^{6\times 6})$
$U_i^{*6\times 1} = [h_i^*, X_i^*, F_i^*, h_{i-1}^*, X_{i-1}^*, F_{i-1}^*]^T$	$U_i^{6\times 1} \sim N_6([\boldsymbol{\mu}_U + \boldsymbol{\delta}_{U,i}]^{6\times 1}, \Sigma_U^{6\times 6}$
Model II: Eq. 5	
$-ch_i^* + X_i^* = R^{1\times 3}V_i^{*3\times 1} = B$	$V_i^{3\times 1} = V_i^{*3\times 1} + \boldsymbol{\delta}_{V,i}^{3\times 1} + E_{V,i}^{3\times 1}$
where $i \geq 1$	$V_i^{*3\times 1} \sim N_3(\boldsymbol{\mu}_V^{3\times 1}, \Sigma_{\lambda V}^{3\times 3})$
$R^{1\times 3} = [-c, 1, 0]$	$E_{V,i}^{3\times 1} \sim N_3(0^{3\times 1}, \Sigma_{EV}^{3\times 3})$
$V_i^{*3\times 1} = [h_i^*, X_i^*, F_i^*]^T$	$V_i^{3\times 1} \sim N_3([\boldsymbol{\mu}_V + \boldsymbol{\delta}_{V,i}]^{3\times 1}, \Sigma_V^{3\times 3})$
Model III: Subtract Eq. 4 at time $i-1$ from Eq. 4 at time i	
$dh_i^* - (b+d)h_{i-1}^* + bh_{i-2}^* + k_{X,2}X_i^* - k_{X,2}X_{i-1}^*$ $- F_i^* + F_{i-1}^* = S^{1\times 9}W_i^{*9\times 1} = 0$	$W_i^{9\times 1} = W_i^{*9\times 1} + \boldsymbol{\delta}_{W,i}^{9\times 1} + E_{W,i}^{9\times 1}$
where $i \geq 2$	$W_i^{*9\times 1} \sim N_9(\boldsymbol{\mu}_W^{9\times 1}, \Sigma_{\lambda W}^{9\times 9})$
$S^{1\times 9} = [d, k_{X,2}, -1, -(b+d), -k_{X,2}, 1, b, 0, 0]$	$E_{W,i}^{9\times 1} \sim N_9(0^{9\times 1}, \Sigma_{EW}^{9\times 9})$
$W_i^{*9\times 1} = [h_i^*, X_i^*, F_i^*, h_{i-1}^*, X_{i-1}^*, F_{i-1}^*, h_{i-2}^*, X_{i-2}^*, F_{i-2}^*]^T$	$W_i^{9\times 1} \sim N_9([\boldsymbol{\mu}_W + \boldsymbol{\delta}_{W,i}]^{9\times 1}, \Sigma_W^{9\times 9})$
Model IV: Substitute Eq. 5 into Eq. 4	
$(d + ck_{X,2})h_i^* - bh_{i-1}^* - F_i^* = G^{1\times 6}Z_i^{*6\times 1}$ $= -\gamma - Bk_{X,2}$	$Z_i^{6\times 1} = Z_i^{*6\times 1} + \boldsymbol{\delta}_{Z,i}^{6\times 1} + E_{Z,i}^{6\times 1}$
where $i \geq 2$	$Z_i^{*6\times 1} \sim N_6(\boldsymbol{\mu}_Z^{6\times 1}, \Sigma_{\lambda Z}^{6\times 6})$
$G^{1\times 6} = [d + ck_{X,2}, 0, -1, -b, 0, 0]$	$E_{Z,i}^{6\times 1} \sim N_6(0^{6\times 1}, \Sigma_{EZ}^{6\times 6})$
$Z_i^{*6\times 1} = [h_i^*, X_i^*, F_i^*, h_{i-1}^*, X_{i-1}^*, F_{i-1}^*]^T$	$Z_i^{6\times 1} \sim N_6([\boldsymbol{\mu}_Z + \boldsymbol{\delta}_{Z,i}]^{6\times 1}, \Sigma_Z^{6\times 6})$

Table 2. Transformed Test Models

Model	Transformed Variable	Distribution
I	$T_{U,i}=Q^{1\times 6}U_i^{6\times 1}$ $=-\gamma+Q^{1\times 6}\delta_{U,i}^{6\times 1}+Q^{1\times 6}E_{U,i}^{6\times 1}$	$T_{U,i}\sim N([-\gamma+Q^{1\times 6}\delta_{U,i}^{6\times 1}],\ \sigma_{TU}^2)$ where $\sigma_{TU}^2=Q^{1\times 6}\ \Sigma_U^{6\times 6}Q^{T\,6\times 1}$
II	$T_{V,i}=R^{1\times 3}V_i^{3\times 1}$ $=-\gamma+R^{1\times 3}\delta_{V,i}^{3\times 1}+R^{1\times 3}E_{V,i}^{3\times 1}$	$T_{V,i}\sim N([B+R^{1\times 3}\delta_{V,i}^{3\times 1}],\ \sigma_{TV}^2)$ where $\sigma_{TV}^2=R^{1\times 3}\ \Sigma_U^{3\times 3}R^{T\,3\times 1}$
III	$T_{W,i}=S^{1\times 9}W_i^{9\times 1}$ $=-\gamma+S^{1\times 9}\delta_{W,i}^{9\times 1}+S^{1\times 9}E_{W,i}^{9\times 1}$	$T_{W,i}\sim N([S^{1\times 9}\delta_{W,i}^{9\times 1}],\ \sigma_{TW}^2)$ where $\sigma_{TW}^2=S^{1\times 9}\ \Sigma_W^{9\times 9}S^{T\,9\times 1}$
IV	$T_{Z,i}=G^{1\times 6}Z_i^{6\times 1}$ $=-\gamma+G^{1\times 6}\delta_{Z,i}^{6\times 1}+G^{1\times 6}E_{Z,i}^{6\times 1}$	$T_{Z,i}\sim N([-\gamma-Bk_{X,2}+G^{1\times 6}\delta_{Z,i}^{6\times 1}],\ \sigma_{TZ}^2)$ where $\sigma_{TZ}^2=G^{1\times 6}\ \Sigma_Z^{6\times 6}G^{T\,6\times 1}$

$$E[T_{U,i}]=E[-\gamma+Q^{1\times 6}\delta_{U,i}^{6\times 1}+Q^{1\times 6}E_{U,i}^{6\times 1}]$$
$$=E[-\gamma]+Q^{1\times 6}E[\delta_{U,i}^{6\times 1}]+Q^{1\times 6}E[E_{U,i}^{6\times 1}]$$
$$=-\gamma+Q^{1\times 6}\delta_{U,i}^{6\times 1},$$
$$\mathrm{Var}(T_{U,i})=Q^{1\times 6}S^{6\times 6}UQT^{6\times 1}=\sigma_{TU}^2,\quad \text{and}$$
$$T_{U,i}\sim N([-\gamma+Q^{1\times 6}\delta_{U,i}^{6\times 1}],\sigma_{TU}^2).\quad (7)$$

The other three models are transformed similarly. The transformation results are presented in Table 2.

Statistical Tests

From the transformed test model we define the model test that attempts to determine if the vector of measured variables $\theta_J(J=\text{I, II, III, or IV})$ is nonzero (i.e., null hypothesis $H_0:\theta_J=0$ vs. alternative hypothesis $H_a:\theta_J\neq 0$). For example, Model I has $\theta_\text{I}=[\delta_h,\ \delta_X,\ \delta_F,\ \gamma]^T$, and from Eq. 6 we propose the following test: reject $H_0:\theta_\text{I}=0$ in favor of $H_a:\theta_\text{I}\neq 0$ if and only if

$$\frac{(T_{U,i})^2}{\sigma_{TU}^2}\geq \chi_{1,\alpha}^2,\quad (8)$$

where $\chi_{1,\alpha}^2$ is the upper (100α)th percentile of the χ^2 distribution with 1 degree of freedom. The power function for this test was derived by Cheng (1994). If the model I test is rejected, we conclude that at least one component of θ_I is not equal to zero. A summary of hypotheses and appropriate conclusions is shown in Table 3.

Global Test

For this particular setting, our GT attempts to determine if any measurement biases, controller bias, or process leaks exist; that is, if $\theta=[\delta_h,\ \delta_X,\ \delta_F,\ \gamma,\ B]^T$ is nonzero. Since none of the four models can test θ, we must test more than one for a complete GT analysis. We chose to test for a nonzero θ_I or θ_II. This is intuitively satisfying because Models I and II are developed from the two basic process equations, unlike for Model III and Model IV.

For a null hypothesis of $H_0:\theta=0$ and an alternative hypothesis of $H_a:\theta\neq 0$, our proposed GT is to reject H_0 in favor of H_a if and only if either the Model I or the Model II test is rejected. The time that the fault occurred (TOC) is taken to be the time at which either null hypothesis was rejected. As in all GTs, this procedure is inadequate to determine the specific fault.

Identification Test Strategy

To identify the specific type of fault, a new identification test strategy (ITS) is proposed. We begin by stating two assumptions: (1) no more than one component of θ is nonzero; (2) once a bias or leak exists, it remains (with the same magnitude) until it is identified and corrected. The first assumption is critical to our identification strategy but the second assumption is made for simplicity. The strategy that we are proposing uses the test statistics that we have developed in Table 2 with the hypothesis given in Table 3, which are used according to the flow chart given in Figure 2 to identify the nonzero δ_h, δ_X, δ_F, γ, or B.

Table 3. Hypothesis Tests for the Four Test Models

Model	H_0 Parameter	Rejected	Conclusions
I	$\theta_\text{I}=[\delta_h,\ \delta_X,\ \delta_F,\ \gamma]^T$	Yes No	At least one component of θ_I is nonzero $\delta_h=\delta_X=\delta_F=\gamma=0$
II	$\theta_\text{II}=[\delta_h,\ \delta_X,\ B]^T$	Yes No	At least one component of θ_II is nonzero $\delta_h=\delta_X=B=0$
III	$\theta_\text{III}=[\delta_h,\ \delta_X,\ \delta_F]^T$	Yes No	At least one component of θ_III is nonzero $\delta_h=\delta_X=\delta_F=0$
IV	$\theta_\text{IV}=[\delta_h,\ \delta_F,\ \gamma,\ B]^T$	Yes No	At least one component of θ_IV is nonzero $\delta_h=\delta_F=\gamma=B=0$

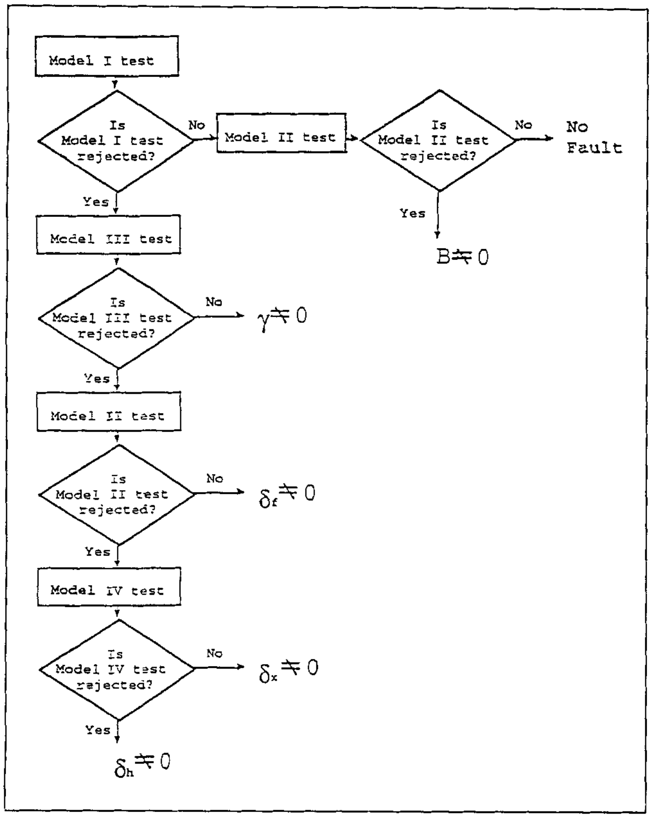

Figure 2. Fault identification testing strategy.

Simulation Study

The relative performance of the proposed fault ITS is demonstrated by a simulation study in which a single fault of a given magnitude was artificially generated at a specified time instant in each simulation run (consisting of 1,000 simulation trials). Testing conditions, including changes in the type of fault, the magnitude of the fault, the true TOC, the level of significance (α), and the sample size N were varied from run to run. The values of the coefficients for the level control process used in this study (Cheng, 1994) are given in Table 4. Specific values were not given to σ_h, σ_X, σ_F (for simplicity) since, for a simulation, the process variability could be captured in the measurement variability.

Four measures of performance are computed to evaluate our ITS: the average type I error (TIE), the overall power (OP), the mean deviation of the TOC estimator (AVGD), and the standard deviation of the TOC estimator (STDD). The expressions for these measures are given below:

$$\text{TIE} = \frac{\text{no. of trials with the type of fault misidentified}}{\text{no. of simulation trials}} \tag{9}$$

$$\text{OP} = \frac{\text{no. of trials with the type of fault correctly identified}}{\text{no. of faults simulated}} \tag{10}$$

$$\text{AVGD} = \sum_{i=1}^{N_T} \frac{(\hat{D}_i - \text{TTOC})}{N_T} \tag{11}$$

Table 4. Parameter Values of Level Control Process

Parameter	Value	Units
$k_{h,1}$	7.2	cm^2/min·cm
$k_{h,2}$	34.78	cm^2/min·cm
$k_{X,2}$	1,156.0	cm^2/min V
k_c	2.0	psi/psi
k_1	0.173	psi/cm
k_2	−0.3	V/psi
a	280.0	cm^2
T	30.0	s
σ_h^2	1.346	cm^2
σ_X^2	0.0144	V^2
σ_F^2	62,500.0	$(cm^3/min)^2$

$$\text{STDD} = \sqrt{\sum_{i=1}^{N_T} \frac{(\hat{D}_i - \text{AVGD})^2}{N_T}}, \tag{12}$$

where the $\hat{D}_i$ are the estimated TOCs, TTOC is the true TOC, and N_T is the number of trials in which the type of fault has been identified correctly. Perfect identification occurs when OP = 1, TIE = 0, AVGD = 0, and STDD = 0.

The results are given in Table 5. The most surprising finding was the sensitivity of ITS to α when sample size $N = 1$. Comparing Table 5a to 5b shows that OP increases and TIE decreases with decreasing α except when the bias magnitude in h' is 10. A possible reason for the exception is that h' is concluded to be biased only if all four null hypotheses are rejected, and the probability of rejecting all four models decreases as α decreases. However, for large enough sample size and/or bias magnitude in h' exceptional performance is still possible. In particular, perfect identification was attained in this study when the bias in h' was 20 with $\alpha = 0.001$. The effect of α for $N = 3$ can be seen by comparing Table 5d to 5f.

The effect of bias magnitude is seen by comparing Table 5a and 5b to Table 5c and 5d. As expected, ITS is more accurate with larger bias magnitudes. The effect of sample size (N) can be seen by comparing Table 5b and 5e. Perfect identification was achieved in our simulations with a sample size of 5. Finally, comparing Table 5a and 5g shows that TTOC did not have significant effect on ITS performance. Therefore, ITS performance improves as α decreases, bias magnitude increases, and N increases, but is not affected by the TTOC.

Conclusions

In this article, we have addressed how a bias measurement, controller bias, or a process leak can be accurately identified in a level control system when the process is in pseudo steady state. We have developed and evaluated a new technique inspired by the work of Rollins and Davis (1992) and Rollins and Devanathan (1993, 1994). Using simulated data, the new technique was determined to be very effective in detecting and identifying the type of fault and its time of occurrence. The main drawback in our technique is that it assumes only one existing fault. However, a fast and accurate method for identifying faults in a simple scale would be advantageous in a fault diagnosis system for a complex chemical plant that

Table 5. Simulation Results

	Type	Mag	TTOC	N	α	OP	TIE	AVGD	STDD
(a)	h	10	10	1	0.001	0.68	0.31	0.00	0.00
	X	8	10	1	0.001	0.94	0.06	0.03	0.75
	F	150	10	1	0.001	0.93	0.07	0.04	0.79
	γ	60	10	1	0.001	0.96	0.04	2.26	6.63
	B	0.6	10	1	0.001	0.93	0.02	3.55	8.09
(b) Effect of α ($N=1$)	h	10	10	1	0.01	0.93	0.06	−0.03	0.40
	X	8	10	1	0.01	0.64	0.36	0.00	0.04
	F	150	10	1	0.01	0.61	0.38	0.38	0.91
	γ	60	10	1	0.01	0.67	0.32	2.62	7.40
	B	0.6	10	1	0.01	0.82	0.17	3.47	8.07
(c) Effect of bias magnitude ($\alpha=0.001$)	h	15	10	1	0.001	0.99	0.01	0.00	0.00
	X	6	10	1	0.001	0.75	0.25	8.63	1.55
	F	100	10	1	0.001	0.58	0.41	0.26	2.36
	γ	40	10	1	0.001	0.65	0.04	6.66	11.38
	B	60	10	1	0.001	0.65	0.02	7.52	12.31
(d) Effect of bias magnitude ($\alpha=0.01$)	h	15	10	1	0.01	1.00	0.00	0.00	0.00
	X	6	10	1	0.01	0.61	0.39	0.21	2.06
	F	100	10	1	0.01	0.55	0.44	0.87	4.54
	γ	40	10	1	0.01	0.63	0.33	4.20	8.60
	B	60	10	1	0.01	0.79	0.17	5.71	10.72
(e) Effect of sample size ($\alpha=0.01$)	h	10	10	3	0.01	1.00	0.00	0.00	0.00
	X	8	10	3	0.01	1.00	0.00	0.00	0.00
	F	150	10	3	0.01	0.99	0.01	0.00	0.00
	γ	60	10	3	0.01	1.00	0.00	0.00	0.03
	B	0.6	10	3	0.01	1.00	0.00	0.02	0.72
(f) Effect of α ($N=3$)	h	10	10	3	0.05	1.00	0.00	0.00	0.00
	X	8	10	3	0.05	0.97	0.03	0.00	0.00
	F	150	10	3	0.05	0.97	0.02	0.00	0.00
	γ	60	10	3	0.05	0.97	0.03	0.00	0.03
	B	0.6	10	3	0.05	0.98	0.02	0.02	0.79
(g) Effect of TTOC	h	10	25	1	0.001	0.71	0.28	−0.04	0.69
	X	8	25	1	0.001	0.93	0.07	0.02	0.62
	F	150	25	1	0.001	0.94	0.06	0.01	0.42
	γ	60	25	1	0.001	0.93	0.03	1.86	4.98
	B	0.6	25	1	0.001	0.93	0.02	2.55	6.28
	B	10	40	1	0.001	0.69	0.30	−0.04	0.57
	X	8	40	1	0.001	0.94	0.06	0.02	0.06
	F	150	40	1	0.001	0.94	0.06	0.01	0.26
	γ	60	40	1	0.001	0.89	0.03	0.81	3.57
	B	0.6	40	1	0.001	0.87	0.02	0.75	5.28

may be broken down into sections that work relatively independently. In addition, it should be possible to extend this approach to coexisting multiple faults. Although we considered only a level process in this article, the ideas of this approach should be applicable to many systems in the process industry. Finally, since this approach is nonrecursive or iterative, it has computational speed superiority over approaches relying on these methods.

Acknowledgments

We are grateful to the National Science Foundation for partial support of this research under grants CTS-9310095 and CTS-9453534.

Notation

B = unknown controller bias
F' = deviation in the input flow rate
F_i' = measured value of F' at the time instant i
F_i^* = unknown true value of F' at time instant i
$G^{6\times 1}$ = constraint vector in Table 1, Model IV
h' = deviation in the tank level
h_i' = measured value of h' at time instant i
h_i^* = unknown true value at time instant i
M = number of time instants
p = number of measured variables
$Q^{1\times 6}$ = constraint vector in Table 1, Model I
$R^{1\times 3}$ = constraint vector in Table 1, Model II
$S^{1\times 9}$ = constraint vector in Table 1, Model III
T = sampling period
$T_{U,i}$ = transformation of measurement model in Table 2, Model I
$T_{V,i}$ = transformation of measurement model in Table 2, Model II
$T_{W,i}$ = transformation of measurement model in Table 2, Model III
$T_{Z,i}$ = transformation of measurement model in Table 2, Model IV
$U_i^{6\times 1}$ = vector of measured values at time instant i in Table 1, Model I
$U_i^{*6\times 1}$ = expected value of $U_i^{6\times 1}$
$V_i^{3\times 1}$ = vector of measured values at time instant i in Table 1, Model II
$V_i^{*3\times 1}$ = expected value of $V_i^{3\times 1}$
$W_i^{9\times 1}$ = vector of measured values at time instant i in Table 1, Model III
$W_i^{*9\times 1}$ = expected value of $W_i^{9\times 1}$
X' = deviation of valve position from steady-state value

X_i' = measured value of X' at time instant i
$X_i^{*\prime}$ = unknown steady-state value of X_i' at time instant i
y_{ji} = measured value of variable j at the ith time instant, Eq. 1
y_{ji}^* = unknown steady-state value of variable j at time instant i, Eq. 1
$Z_i^{6\times 1}$ = vector of measured values at time instant i in Table 1, Model IV
$Z_i^{*6\times 1}$ = expected value of $Z_i^{6\times 1}$

Greek letters

γ = magnitude of process leak (unknown)
δ_j = magnitude of measurement bias for variable j, Eq. 1
δ_h = magnitude of measurement bias h'
δ_x = magnitude of measurement bias of X'
δ_F = magnitude of measurement bias of F'
$\boldsymbol{\delta}_{U,i}^{6\times 1}$ = vector of measurement bias in Table 1, Model I
$\boldsymbol{\delta}_{V,i}^{3\times 1}$ = vector of measurement bias in Table 1, Model II
$\boldsymbol{\delta}_{W,i}^{9\times 1}$ = vector of measurement bias in Table 1, Model III
$\boldsymbol{\delta}_{Z,i}^{6\times 1}$ = vector of measurement bias in Table 1, Model IV
ϵ_{ji} = random error of variable j at the ith time instant, Eq. 1
$\boldsymbol{E}_{U,i}^{6\times 1}$ = vector of measurement random errors in Table 1, Model I
$\boldsymbol{E}_{V,i}^{3\times 1}$ = vector of measurement random errors in Table 1, Model II
$\boldsymbol{E}_{W,i}^{9\times 1}$ = vector of measurement random errors in Table 1, Model III
$\boldsymbol{E}_{Z,i}^{6\times 1}$ = vector of measurement random errors in Table 1, Model IV
μ_j = steady-state value of variable j, Eq. 1
$\boldsymbol{\mu}_U^{6\times 1}$ = vector of expected values in Table 1, Model I
$\boldsymbol{\mu}_V^{3\times 1}$ = vector of expected values in Table 1, Model II
$\boldsymbol{\mu}_W^{9\times 1}$ = vector of expected values in Table 1, Model III
$\boldsymbol{\mu}_Z^{6\times 1}$ = vector of expected values in Table 1, Model IV
λ_{ji} = process deviation of variable j from μ_j at the ith time instant, Eq. 1
σ^2 = used for variances
Σ = used for variance-covariance matrices

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Manuscript received Oct. 6, 1994, and revision received Oct. 13, 1995.